

Kenmerk: EWI2019/TW/DMMP/MU/Mod7/Exam2

Exam 2, Module 7, Codes 201400483 & 201800141

Discrete Structures & Efficient Algorithms

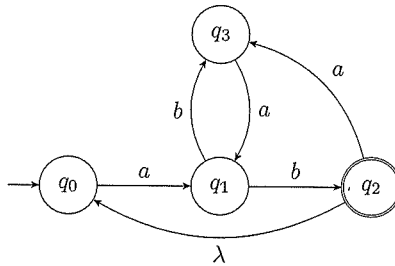
Friday, March 29, 2019, 13:45 - 15:45

All answers need to be motivated. No calculators. You are allowed to use a handwritten cheat sheet (A4, both sides).

This second exam of Module 7 consists of the Languages & Machines part only, and is a 2h exam. The total is 50 points. Your exam grade is the maximum of 1 and the total number of points divided by 5, rounded to one digit.

Languages & Machines

1. (11 points) Consider the following NFA with λ -steps M (only q_2 is accepting):



- First eliminate state q_3 , adding new transitions labeled with regular expressions, to preserve the accepted language. Show the resulting "expression graph".
 - Continue the construction, by eliminating q_1 as well, and read off a regular expression E with $\mathcal{L}(E) = \mathcal{L}(M)$.
 - Provide the λ -closure and input-transition function of the automaton M in a table.
 - Transform the automaton M in a systematic manner to a (possibly incomplete) DFA.
2. (9 points) Consider the definitions of the following languages over $\Sigma = \{a, b\}$:
- Language $L_1 := \{a^{3i} b^{j+1} \mid 0 \leq i \text{ and } 0 \leq j\}$
 - Language $L_2 := \{b^i a^j b^i \mid 0 \leq i \text{ and } 0 \leq j\}$
 - Language L_3 is the set of words over Σ that can be found in the Oxford English Dictionary.
 - Language L_4 is an (arbitrary) infinite language

Indicate for each of the following languages if they are regular or not. Motivate your answers, either by a proof or a construction.

- Language L_1
- Language L_2
- Language $\overline{L_4} \cap L_3$

3. (5 points) Which variables can be derived by chain rules from C in the following grammar G_1 ? Provide an equivalent grammar without chain rules.

$$G_1 = \begin{cases} S \rightarrow AB \mid C \\ A \rightarrow aA \mid B \\ B \rightarrow bB \mid C \\ C \rightarrow cC \mid a \mid A \end{cases}$$

4. (5 points) Provide a *regular* grammar, equivalent to the following context-free grammar G_2 :

$$G_2 = \begin{cases} S \rightarrow BS \mid AB \\ A \rightarrow aAa \mid \lambda \\ B \rightarrow b \mid Bb \end{cases}$$

5. (5 points) Consider the context-free language $L = \{a^i b^j \mid i > j > 0\}$. Give a PDA (pushdown automaton) for L . Provide a *short* explanation.
6. (9 points, every wrong answer costs 2 points) Indicate for each of the following statements if they are TRUE or FALSE. (No explanation required).
- Every contextfree grammar (CFG) has a Turing Machine (TM) accepting the same language.
 - Every contextfree grammar (CFG) has an equivalent extended PDA with two states.
 - The class of contextfree languages is closed under complement.
 - The class of contextfree languages is closed under union.
 - To every PDA there exists an equivalent deterministic PDA.
 - To every TM there exists an equivalent deterministic TM.
 - The language of (encoded) terminating Turing Machines is not recursive, but it is recursively enumerable.
 - Given a grammar G in Chomsky Normal Form and a word w , one can decide in polynomial time whether $w \in \mathcal{L}(G)$.
 - The class of recursive languages is closed under complement.
7. (6 points) Which language is *decided* by the following Turing Machine (only q_3 is accepting)? Explain your answer *shortly*.

